

Cosmological constant in SUGRA models and the multiple point principle

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Abstract

The tiny order of magnitude of the cosmological constant is sought to be explained in a model involving the following ingredients: supersymmetry breaking in N=1 supergravity and the multiple point principle. We demonstrate the viability of this scenario in the minimal SUGRA model.

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1. Introduction

As is well known, cosmology yields strong arguments against the Standard Model (SM). Although the SM describes perfectly the major part of all experimental data measured in earth based experiments, it does not provide any reliable candidate for dark matter. Another puzzle of modern cosmology is a tiny density of energy spread all over the Universe (the cosmological constant), which is responsible for its acceleration. At first glance this energy density (Λ) should be of the order of the Planck, or possibly the electroweak, scale to the fourth power; however a fit to the recent data shows that $\Lambda \sim 10^{-123} M_{Pl}^4 \sim 10^{-55} M_Z^4$ [1]. The smallness of the cosmological constant should be considered as a fine-tuning problem, for which new theoretical ideas must be employed to explain the enormous cancellations between the contributions of different condensates to the cosmological constant.

Unfortunately the cosmological constant problem can not be resolved in any available generalization of the SM. An exact global supersymmetry (SUSY) ensures zero value for the energy density at the minimum of the potential of the scalar fields. But in the exact SUSY limit bosons and fermions from one chiral multiplet get the same masses. Soft supersymmetry breaking, which guarantees the absence of superpartners of observable fermions in the 100 GeV range, does not protect the cosmological constant from an electroweak scale mass and the fine-tuning problem is re-introduced.

In this article we propose the multiple point principle (MPP) [2] as a basic principle to explain the size of the cosmological constant. MPP postulates that in Nature as many phases as possible, which are allowed by the underlying theory, should coexist. On the phase diagram of the theory it corresponds to the special point – the multiple point – where many phases meet. The vacuum energy densities of these different phases are degenerate at the multiple point. In other words Nature should adjust all the couplings of the SM (or any other model) such that a number of degenerate vacua are realized. The MPP relations between coupling constants can arise dynamically. For example a mild form of locality breaking in quantum gravity, due to baby universes [3] say, is expected to precisely fine-tune the couplings so that indeed several phases with degenerate vacua coexist. Another possible origin for MPP could be a symmetry. Supersymmetry is the best candidate for this role because all global vacua in SUSY models are degenerate. Moreover the SUSY scalar potential often contains a few flat directions with zero vacuum energy.

The idea of the multiple point principle was applied to the pure SM, by postulating that the Higgs effective potential has two rings of minima in the Mexican hat with the same vacuum energy density [4] (the effective potential depends only on the Higgs field

norm and has two minima in it). The radius of the little ring is at the electroweak vacuum expectation value of the Higgs field, while the radius of the big one was assumed to be near the Planck scale ($M_{Pl} \approx 10^{19}$ GeV). These two assumptions lead to rather precise predictions for the top quark (pole) and Higgs boson masses [4]

$$M_t = 173 \pm 4 \text{ GeV}, \quad M_H = 135 \pm 9 \text{ GeV}. \quad (1)$$

In the present article MPP is used, in conjunction with supersymmetric models, to deduce a size for the cosmological constant to be compared with the value obtained by astrophysical observations. We shall do this by assuming *a priori* the existence of a supersymmetric phase in flat (Minkowski) space, in addition to the phase in which we live. Since the vacuum energy density (cosmological constant) of supersymmetric states in flat Minkowski space is just zero, the cosmological constant problem is thereby solved to first approximation by assumption. Now the strategy is to estimate the SUSY breaking energy contribution in this most supersymmetric phase, which is the only contribution to the cosmological constant in this case and, by virtue of MPP, to assign the found value to all other phases and especially to the one in which we live.

However such a procedure immediately raises the question as to why the most supersymmetric phase is taken, among all the various ones, to get its value for the cosmological constant transferred via MPP to all the other phases. The suggested answer is that one should choose, for this purpose, the phase with the smallest SUSY breaking and thus lowest cosmological constant to be the decisive phase. It comes from the philosophy that it is easier for MPP to tune some cancellation, in order to make a quantity small, than it is to get its value strongly enhanced.

Of course we do not solve the cosmological constant problem entirely in this work. Indeed there is a murky point in the suggested procedure. In order to have a tiny value of the cosmological constant in the phase where the supersymmetry is broken severely, we must call upon supergravity (SUGRA). In this case a hidden sector can give an additional contribution to the total energy density cancelling ones from other sources (like electroweak symmetry breaking in our phase for example). At the same time, even in the vacuum where local supersymmetry remains intact, the total energy density tends to be huge and negative. It makes our initial assumption concerning the existence of a phase with global SUSY in flat Minkowski space rather artificial. An extra fine-tuning is required to obtain a viable solution of this type and corresponds to searching for only a partial solution of the cosmological constant problem. The aim of this paper is to calculate the deviation from zero cosmological constant, once our initial assumption is accepted.

This article is organized as follows: in the next section we describe the structure of ($N = 1$) SUGRA models and discuss the mechanism of supersymmetry breaking; we

formulate our MPP supergravity model in section 3 and present some numerical estimates of the vacuum energy density in section 4. Our results are summarized in section 5.

2. From supergravity to SM

SUSY models clear the way to the unification of gauge interactions with gravity. Such unification is carried out in the framework of SUGRA models. Simplest ($N = 1$) supersymmetric models correspond to ($N = 1$) supergravity. The full ($N = 1$) SUGRA Lagrangian [5] is specified in terms of an analytic gauge kinetic function $f_a(\phi_M)$ and a real gauge-invariant Kähler function $G(\phi_M, \phi_M^*)$, which depend on the chiral superfields ϕ_M . The function $f_a(\phi_M)$ determines the kinetic terms for the fields in the vector supermultiplets and the gauge coupling constants $Ref_a(\phi_M) = 1/g_a^2$, where the index a designates different gauge groups. The Kähler function is a combination of two functions

$$G(\phi_M, \phi_M^*) = K(\phi_M, \phi_M^*) + \ln |W(\phi_M)|^2, \quad (2)$$

where $K(\phi_M, \phi_M^*)$ is the Kähler potential whose second derivatives define the kinetic terms for the fields in the chiral supermultiplets. $W(\phi_M)$ is the complete analytic superpotential of the considered SUSY model. In this article standard supergravity mass units are used: $\frac{M_{Pl}}{\sqrt{8\pi}} = 1$.

Experimentally, of course, SUSY can not be an exact symmetry at low energies and has to be broken in such a way that quadratic divergences are not induced (so-called soft SUSY breaking). In SUGRA models local supersymmetry breaking happens in a hidden sector, which contains singlet superfields (h_m) under the SM $SU(3) \times SU(2) \times U(1)$ gauge group. These hidden superfields are introduced by hand in the simplest models.

In theoretically reliable SUGRA models the form of the Kähler function and the structure of the hidden sector are fixed by an underlying renormalizable or even finite theory. Nowadays the best candidate for the ultimate theory is $E_8 \times E_8$ (ten dimensional) heterotic superstring theory [6]. The strong coupling limit in this theory can be described by eleven dimensional supergravity on a manifold with two ten-dimensional boundaries (M-theory) [7]. Gauge multiplets of each E_8 gauge group are localized on a separate boundary and interact with multiplets of the other E_8 by virtue of gravitational forces. Compactification of the extra dimensions on a Calabi-Yau manifold leads to an effective supergravity and results in the breaking of one E_8 to E_6 or its subgroups, which play the role of gauge symmetries in the observable sector. Multiplets of the remaining E_8 belong to the hidden part of the considered theory. Although all hidden sector multiplets can give rise to violation of supersymmetry, the minimal possible SUSY-breaking sector in string models involves dilaton (S) and moduli (T_m) superfields. The number of moduli varies

from one string model to another. But dilaton and moduli fields are always present in four-dimensional heterotic superstrings, because S is related with the gravitational sector while vacuum expectation values of T_m determine the size and shape of the compactified space.

After integration over hidden sector fields the superpotential of SUGRA models generally looks like (see for example [8])

$$W(\phi_M) = W^{(tree)}(\phi_M) + W^{(ind)}(\phi_M), \quad (3)$$

where

$$W^{(tree)}(\phi_M) = \frac{1}{6} Y'_{\alpha\beta\gamma}(h_m) C^\alpha C^\beta C^\gamma + \dots \quad (4)$$

is a classical superpotential that depends on hidden h_m (dilaton and moduli) and observable C^α superfields. Generally supersymmetric mass terms are assumed to be absent in the classical part of the superpotential. They may be induced by non-perturbative corrections that summarize the effects of integrating out the hidden sector [8]:

$$W^{(ind)}(\phi_M) = \hat{W}(h_m) + \frac{1}{2} \mu'_{\alpha\beta}(h_m) C^\alpha C^\beta + \dots \quad (5)$$

Expanding the full Kähler potential in powers of observable fields C^α , we have [8]-[9]

$$K = \hat{K}(h_m, h_m^*) + \tilde{K}_{\bar{\alpha}\beta}(h_m, h_m^*) C^{*\bar{\alpha}} C^\beta + \left[\frac{1}{2} Z_{\alpha\beta}(h_m, h_m^*) C^\alpha C^\beta + h.c. \right] + \dots, \quad (6)$$

The dots in formulas (4)–(6) stand for higher order terms whose coefficients are suppressed by negative powers of M_{Pl} . $\hat{W}(h_m)$ and $\hat{K}(h_m, h_m^*)$ are the superpotential and the Kähler potential of the hidden sector respectively. Notice that the coefficients $Y'_{\alpha\beta\gamma}$, $\mu'_{\alpha\beta}$, $\tilde{K}_{\bar{\alpha}\beta}$ and $Z_{\alpha\beta}$ in the expansions (4)–(6) depend on the hidden sector fields in general. The bilinear terms associated with $\mu'_{\alpha\beta}$ and $Z_{\alpha\beta}$ are often forbidden by gauge invariance. However their appearance destroys the Z_3 discrete symmetry that leads to the domain wall problem [10] and provides a viable solution for the so-called μ problem, in the context of the minimal supersymmetric standard model (MSSM) [11].

The SUGRA scalar potential can be presented as a sum of F - and D -terms $V_{SUGRA}(\phi_M, \phi_M^*) = V_F(\phi_M, \phi_M^*) + V_D(\phi_M, \phi_M^*)$, where the F -part is given by [5],[12]

$$\begin{aligned} V_F(\phi_M, \phi_M^*) &= e^G (G_M G^{M\bar{N}} G_{\bar{N}} - 3), \\ G_M &\equiv \partial_M G \equiv \partial G / \partial \phi_M, \quad G_{\bar{M}} \equiv \partial_{\bar{M}} G \equiv \partial G / \partial \phi_M^*, \\ G_{\bar{N}M} &\equiv \partial_{\bar{N}} \partial_M G = \partial_{\bar{N}} \partial_M K \equiv K_{\bar{N}M}. \end{aligned} \quad (7)$$

The matrix $G^{M\bar{N}}$ is the inverse of the Kähler metric $K_{\bar{N}M}$. If, at the minimum of the scalar potential, hidden sector fields acquire vacuum expectation values so that at least one of their auxiliary fields

$$F^M = e^{G/2} G^{M\bar{P}} G_{\bar{P}} \quad (8)$$

is non-vanishing, then local SUSY is spontaneously broken. At the same time a massless fermion with spin $1/2$ – the goldstino, which is a combination of the fermionic partners of the hidden sector fields giving rise to the breaking of SUGRA, is swallowed by the gravitino that becomes massive

$$m_{3/2} = \langle e^{G/2} \rangle .$$

This phenomenon is called the super-Higgs effect [13].

Since the superfields of the hidden sector interact with the observable ones only by means of gravity, they are decoupled from the low energy theory. The only signal they produce are a set of terms that break the global supersymmetry of the low-energy effective Lagrangian of the observable sector in a soft way [14]–[15]. The set of soft SUSY breaking parameters includes: gaugino masses M_a , masses of scalar components of observable superfields m_α , trilinear $A_{\alpha\beta\gamma}$ and bilinear $B_{\alpha\beta}$ scalar couplings associated with Yukawa couplings and μ -terms in the superpotential of the considered SUSY model [16]. Using the explicit form of the SUGRA scalar potential (7) and the expansion of the Kähler function in terms of observable superfields (4)–(6), one can compute soft SUSY breaking terms at the Planck or Grand Unification scale. They are obtained by substituting vacuum expectation values for the hidden sector fields h_m and corresponding auxiliary fields F^m , and taking the flat limit [17] where $M_{Pl} \rightarrow \infty$ but $m_{3/2}$ is kept fixed. Then one is left with a global SUSY Lagrangian plus the soft SUSY breaking terms listed above. All non-renormalizable terms can be omitted, since they are suppressed by inverse powers of M_{Pl} . Choosing the Kähler metric of the observable sector in the diagonal form $\tilde{K}_{\bar{\alpha}\beta} = \tilde{K}_\alpha \delta_{\bar{\alpha}\beta}$ to avoid dangerous flavour-changing neutral current (FCNC) transitions and assuming that, at the minimum of the SUGRA scalar potential, the value of the cosmological constant equals zero $\langle V(h_m) \rangle = 0$, one finds [8]–[9]

$$\begin{aligned} m_\alpha^2 &= m_{3/2}^2 - F^{\bar{m}} F^n \partial_{\bar{m}} \partial_n \ln \tilde{K}_\alpha , \\ A_{\alpha\beta\gamma} &= F^m \left[\hat{K}_m + \partial_m \ln Y'_{\alpha\beta\gamma} - \partial_m \ln(\tilde{K}_\alpha \tilde{K}_\beta \tilde{K}_\gamma) \right] , \\ M_a &= \frac{1}{2} (Re f_a)^{-1} F^m \partial_m f_a . \end{aligned} \tag{9}$$

As usual the D-term contributions to SUSY breaking are neglected. Explicit expressions for the bilinear scalar couplings $B_{\alpha\beta}$ are not given here, because they depend significantly on the mechanism of μ -term generation (see [9]).

The first term in the formula for m_α^2 gives a universal positive contribution to all soft scalar masses that, in general, allows us to make scalar particles heavier than their fermionic partners. The size of all soft SUSY breaking terms is characterized by the gravitino mass scale. Therefore the gravitino mass should not be very large, since the

soft masses of the Higgs bosons have to be of the order of the electroweak scale to ensure a correct pattern for the $SU(2) \times U(1)$ symmetry breaking. A huge mass hierarchy ($m_{3/2} \ll M_{Pl}$) can appear due to a non-perturbative source of local supersymmetry breaking in the hidden sector gauge group [18].

3. Multiple Point Principle in SUGRA models

Let us now consider SUGRA models which obey the multiple point principle. This means that there must be two or even more degenerate vacua in the considered models. In the Standard Model and its renormalizable extensions the MPP conditions are attained by adjusting arbitrary coupling constants. As mentioned in section 2, in SUGRA models there are two arbitrary functions that should be fixed via MPP, in the same way as the coupling constants in renormalizable theories, resulting in a set of degenerate vacua.

As described above a common paradigm implies that, at one minimum of the scalar potential (7), local supersymmetry is broken leading to the appearance of soft terms in the effective Lagrangian of the observable sector. Further we will treat this vacuum as the physical one, which is realized in Nature, and denote the appropriate vacuum expectation values of hidden fields as $h_m^{(1)}$.

However MPP inspired SUGRA models may have another minimum of the scalar potential with the same energy density, where the supersymmetry in the hidden sector is unbroken. Moreover we assume that, in this second vacuum, the low energy limit of the considered theory is described by a pure supersymmetric model in flat Minkowski space. As discussed in the introduction, the last requirement represents an extra fine-tuning because in general the cosmological constant in SUGRA models is huge and negative. To show this, let us suppose that, the Kähler function has a stationary point $\phi_M = \phi_M^0$, where $G_M = 0$. Then it is easy to check that this point is also an extremum of $V_{SUGRA}(\phi_M, \phi_M^*)$. Since, according to Eq.(8), the auxiliary fields F^M are “proportional” to the G_M , they vanish in its vicinity and local supersymmetry remains intact. At the same time the energy density is huge and negative. While all D-terms go to zero near the extremum point of $G(\phi_M, \phi_M^*)$, the last term in the brackets of Eq.(7) for $V_F(\phi_M, \phi_M^*)$ gives a finite and negative contribution to the total density of energy. Thus the cosmological constant in such SUGRA models is less than or equal to $-3e^{G(\phi_M^0, \phi_M^{0*})}$.

On the other hand, in flat Minkowski space the energy density of any supersymmetric vacuum state is exactly zero. The effective description of the second vacuum, in terms of a supersymmetric one, is supposed to be valid down to very low energies ($E \ll M_Z$) in the MPP inspired SUGRA model. Thus all soft SUSY breaking terms induced into the observable sector must vanish (with much higher accuracy than in the physical vacuum)

and particles from a single supermultiplet will have the same mass. Since in the SUSY limit the graviton and gravitino are massless in the flat space-time approximation, one obtains an additional constraint on the value of the superpotential of the hidden sector

$$\langle \hat{W}(h_m^{(2)}) \rangle = 0 \quad (10)$$

where $h_m^{(2)}$ denote vacuum expectation values of the hidden sector fields in the second vacuum. Equation (10) is nothing other than the extra fine-tuning in our model that corresponds to giving up the complete solution of the cosmological constant problem.

If condition (10) is fulfilled then the last term in the brackets of Eq.(7), which led to the negative energy density, vanishes. Taking into account that the Kähler metric of the hidden sector is positive definite, one can prove in this case that the absolute minimum of the scalar potential (7) is achieved when

$$\left. \frac{\partial \hat{W}(h_m)}{\partial h_k} \right|_{h_m=h_m^{(2)}} = 0. \quad (11)$$

Together with the superpotential of the hidden sector and its derivatives, the energy density of the second vacuum and the auxiliary fields F^M go to zero, verifying that supersymmetry really is unbroken.

In order to demonstrate how the conditions (10) and (11) work, let us consider a particular example. For the sake of simplicity, we restrict our consideration to the minimal SUGRA model [14], [17], [19] with Kähler potential

$$K(\phi_M, \phi_M^*) = \sum_m h_m h_m^* + \sum_\alpha |C^\alpha|^2 \quad (12)$$

which results in canonical kinetic terms in the supergravity Lagrangian. A canonical choice for the kinetic function $f_a(h_m) = \text{const}$ corresponds to $M_a = 0$. Therefore we assume a mild dependence of $f_a(h_m)$ on the hidden fields, so that the gauge couplings in the physical and supersymmetric vacua do not differ by much, i.e. $|f_a(h_m^{(1)}) - f_a(h_m^{(2)})| \ll f_a(h_m^{(1)})$.

Because the Kähler metric $K_{\bar{N}M}$ and its inverse are diagonal, an explicit form for the SUGRA scalar potential of the hidden sector can be easily found:

$$V_F^{hid}(h_m, h_m^*) = e^{\hat{K}(h_m, h_m^*)} \left(\sum_k \left| \frac{\partial \hat{W}(h_m)}{\partial h_k} + h_k^* \hat{W}(h_m) \right|^2 - 3 |\hat{W}(h_m)|^2 \right). \quad (13)$$

Although in principle the potential (13) takes positive as well as negative values near the second minimum, where supersymmetry is preserved, the energy density is always larger than or equal to zero:

$$\langle V_F^{hid}(h_m^{(2)}) \rangle_{SUSY} = e^{\hat{K}(h_m, h_m^*)} \left(\sum_k \left| \frac{\partial \hat{W}(h_m)}{\partial h_k} \right|^2 \right) \Big|_{h_m=h_m^{(2)}}, \quad (14)$$

while $h_m^{(2)}$ satisfy equations for extrema

$$\sum_k \left(\frac{\partial \hat{W}(h_m)}{\partial h_k} \right)^* \left[\frac{\partial^2 \hat{W}(h_m)}{\partial h_k \partial h_n} + h_n^* \frac{\partial \hat{W}(h_m)}{\partial h_k} + h_k^* \frac{\partial \hat{W}(h_m)}{\partial h_n} \right] = 0. \quad (15)$$

In the minimization conditions (15) we put $\hat{W}(h_m) = 0$. The index n varies from 1 to N , where N is the number of scalar fields in the hidden sector which acquire non-zero vacuum expectation values. From the equations that determine the position of the stationary point of the SUGRA scalar potential (15) and the expression for the energy density in the supersymmetric vacuum (14), it becomes clear that the deepest minimum is reached when the conditions (11) are satisfied and the value of the scalar potential (13) equals zero.

In the instance when the hidden sector contains only one singlet superfield, the simplest superpotential that suits MPP is

$$\hat{W}(S) = m_0(S + \beta)^2. \quad (16)$$

If the parameter $\beta = -\sqrt{3} + 2\sqrt{2}$, the SUGRA scalar potential possesses two degenerate minima with zero energy density at the classical level. The appropriate hidden scalar potential and superpotential as a function of the scalar component of the superfield S are shown in Figs. 1a and 1b. For large $|S| \gtrsim 1$ the SUGRA potential grows rapidly because of the exponential factor $e^{|S|^2}$ that arises due to the first term in (12). There are three extremum points in the scalar potential. The left minimum coincides with the stationary point of the superpotential (16), where it vanishes, so that supersymmetry is unbroken. The right minimum is attained for $\langle S \rangle = S_0 = \sqrt{3} - \sqrt{2}$. In this vacuum, the gravitino gets a mass $m_{3/2} = 1.487 \cdot m_0$ and the set of soft SUSY breaking terms is generated:

$$m_\alpha^2 = m_{3/2}^2, \quad A_{\alpha\beta\gamma} = (3 - \sqrt{6})m_{3/2}. \quad (17)$$

To obtain these results, the explicit expressions for the Kähler potential (12) and superpotential (16) were substituted in formulas (2), (8) and (9), where the field S was replaced by its vacuum expectation value S_0 . The predictions for the gaugino masses M_a are not given here, since we do not specify the dependence of the kinetic function on the hidden field S .

More complex structure in the hidden sector superpotential can lead to a scalar potential that has a few vacua in which the supersymmetry of the full $N = 1$ SUGRA Lagrangian is exact or only spontaneously broken. The MPP requires the degeneracy of all the vacua or at least the deepest physical and supersymmetric ones. If the hidden sector involves more than one superfield, SUGRA models may possess so-called vacuum valleys or flat directions. Then the most preferable situation, from an MPP believer's

point of view, arises when many vacua or flat directions that might be supersymmetric or not have the same energy density. However, having one vacuum obeying the relations (10) and (11) means the existence of just one extra phase degenerate with our own; this is only a beginning or a necessary condition for MPP in the minimal SUGRA models. In general MPP means that there is a number of degenerate minima $h_m^{(i)}$:

$$V(h_m^{(1)}, h_m^{(1)*}) = V(h_m^{(i)}, h_m^{(i)*}),$$

$$\left. \frac{\partial V(h_m, h_m^*)}{\partial h_k} \right|_{h_m=h_m^{(1)}} = \left. \frac{\partial V(h_m, h_m^*)}{\partial h_k} \right|_{h_m=h_m^{(i)}} = 0, \quad (18)$$

Here $V(h_m, h_m^*)$ should be identified with the full SUGRA scalar potential $V_{SUGRA}(h_m, h_m^*)$ or with its F-part $V_F(h_m, h_m^*)$ if all hidden sector fields are singlets.

4. The value of the cosmological constant

In principle, the supersymmetry that remains intact in the second vacuum can be broken dynamically at low energies (for recent reviews see [20]–[21]). Indeed, even in the pure MSSM, the beta function of the strong gauge coupling constant exhibits asymptotically free behaviour ($b_3 = -3$). Since in the minimal SUGRA model the kinetic function does not depend on the hidden superfields ($f_a(h_m) = \text{const}$), the values of the gauge couplings at the unification scale and their running down to the scale $M_S \simeq m_{3/2}$ are the same in both vacua. Below the scale M_S all superparticles in the physical vacuum decouple and the corresponding beta function changes ($\tilde{b}_3 = -7$). Using the value of $\alpha_3^{(1)}(M_Z) \approx 0.118 \pm 0.003$ [22] and matching condition $\alpha_3^{(2)}(M_S) = \alpha_3^{(1)}(M_S)$, one finds the strong coupling in the second vacuum

$$\frac{1}{\alpha_3^{(2)}(M_S)} = \frac{1}{\alpha_3^{(1)}(M_Z)} - \frac{\tilde{b}_3}{4\pi} \ln \frac{M_S^2}{M_Z^2}. \quad (19)$$

Here $\alpha_3^{(1)}$ and $\alpha_3^{(2)}$ are the values of the strong gauge couplings in the physical and second minima of the SUGRA scalar potential.

At the scale Λ_{SQCD} , where the supersymmetric QCD interaction becomes strong in the second vacuum

$$\Lambda_{SQCD} = M_S \exp \left[\frac{2\pi}{b_3 \alpha_3^{(2)}(M_S)} \right] \quad (20)$$

the supersymmetry may be broken due to non-perturbative effects. If instantons generate a repulsive superpotential [20], [23] which lifts and stabilizes the vacuum valleys in the scalar potential, then a generalized O’Raifeartaigh mechanism can take place inducing a non-zero value for the cosmological constant

$$\Lambda \sim \Lambda_{SQCD}^4. \quad (21)$$

In Fig.2 the dependence of Λ_{SQCD} on the SUSY breaking scale M_S is examined. Because $\tilde{b}_3 < b_3$ the QCD gauge coupling below M_S is larger in the physical minimum than in the second one. Therefore the value of Λ_{SQCD} is much lower than in the Standard Model and diminishes with increasing M_S . For the pure MSSM it varies from $10^{-25}M_{Pl}$ to $10^{-30}M_{Pl}$, when M_S grows from 100 GeV to 1000 TeV. From rough estimates of the energy density (21), it can be easily seen that $\Lambda_{SQCD} = 10^{-31}M_{Pl}$ gives the measured value of the cosmological constant. If MSSM is supplemented by an additional pair of $5 + \bar{5}$ multiplets then Λ_{SQCD} of the required size can be reproduced even for $M_S = 100 \div 1000$ GeV.

Achieving the SUSY breaking at the scale Λ_{SQCD} is actually not at all easy. The discussion is different depending on whether the number of flavours N_f is larger or smaller than the number of colours N_c . In the MSSM and its simplest extensions where $N_c = 3$ and $N_f = 6$ the generated superpotential has a polynomial form [21], [24]. The absolute minimum of the SUSY scalar potential is then achieved when all the superfields, including their F- and D-terms, acquire zero vacuum expectation values preserving supersymmetry. This result throws some doubt on our scenario for a tiny cosmological constant, which is based on Eq. (21).

Another method of breaking SUSY is by the appearance of gaugino condensation $\bar{\lambda}_a \lambda_a$. The gaugino condensation itself does not lead to the spontaneous breakdown of global supersymmetry [25]. But if a non-trivial dependence of the gauge kinetic function on the hidden sector fields is assumed then the corresponding auxiliary fields

$$F^i = e^{G/2} G^{i\bar{j}} G_{\bar{j}} - \frac{1}{4} G^{i\bar{j}} \frac{\partial f(h_m)}{\partial h_j} \bar{\lambda}_a \lambda_a + \dots \quad (22)$$

get an extra contribution which is proportional to $\langle \bar{\lambda}_a \lambda_a \rangle \simeq \Lambda_{SQCD}^3$ resulting in local supersymmetry breaking [18] and a non-zero vacuum energy density

$$\Lambda \sim \frac{\Lambda_{SQCD}^6}{M_{Pl}^2}. \quad (23)$$

Unfortunately the gaugino condensation is not likely to occur if $N_f > N_c$.

However the above disappointing facts concerning dynamical SUSY breaking were revealed in the framework of pure supersymmetric QCD, where all Yukawa couplings were supposed to be small or even absent. At the same time the t-quark Yukawa coupling in the MSSM is of the same order of magnitude as the strong gauge coupling at the electroweak scale. Therefore it can change the above results drastically. We plan to continue our investigations of supersymmetry breaking in the pure MSSM and its extensions.

5. Conclusions

In the present article we have applied the multiple point principle assumption to ($N = 1$) supergravity. At first we reviewed the structure of the ($N = 1$) SUGRA Lagrangian and local supersymmetry breaking via the hidden sector. Explicit expressions for the soft SUSY breaking parameters in terms of the Kähler and gauge kinetic functions were also collected. The MPP inspired SUGRA model we considered implies that the corresponding scalar potential contains at least two degenerate minima. In one of them local supersymmetry is broken in the hidden sector at the high energy scale ($\sim 10^{10} - 10^{12}$ GeV), inducing a set of soft SUSY breaking terms for the observable fields. In the other vacuum the low energy limit of the considered theory is described by a pure supersymmetric model in flat Minkowski space. This second minimum is realized if the superpotential of the hidden sector has an extremum point where it goes to zero. The stationary point of the superpotential coincides with the position of the second minimum of the SUGRA scalar potential. The energy density and all auxiliary fields F^M of the hidden sector vanish in the second vacuum preserving supersymmetry. The simplest SUGRA model, where the MPP conditions are satisfied, has been discussed and the predictions for the soft masses and trilinear scalar couplings have been obtained.

Non-perturbative effects in the observable sector can give rise to supersymmetry breakdown in the second vacuum (phase). In this case the value of the energy density is determined by the scale where the gauge interactions become strong. Numerical estimates have been carried out in the framework of the pure MSSM. They reveal that the corresponding scale is naturally low ($\Lambda_{SQCD} \simeq 10^{-30} - 10^{-25} M_{Pl}$) providing a tiny energy density of the second phase. The crucial idea is then to use MPP to transfer the energy density or cosmological constant from this second vacuum into all other vacua, especially into the physical one in which we live. In such a way we have suggested an explanation of why the observed value of the cosmological constant has the tiny value it has.

The trouble with the considered approach is that the dynamical breakdown of supersymmetry looks rather questionable, in models which involve QCD with more flavours than colours (as in the SM and its simplest SUSY extensions). However the strong interaction between the Higgs and t-quark superfields in the superpotential, which has always been ignored in previous considerations, could play a decisive role. We intend to study this problem in more detail.

Acknowledgements

The authors are grateful to S.Bludman, D.Kazakov, O.Kancheli, V.Novikov, L.Okun,

M.Shifman, K.A.Ter-Martirosyan, M.Vysotsky and P.Zerwas for fruitful discussions and A.Anisimov and M.Trusov for useful comments. R.Nevzorov and H.B.Nielsen are indebted to the DESY Theory Group for hospitality extended to them in 2002 when this project was started. R.Nevzorov is also grateful to Alfred Toepfer Stiftung for the scholarship and for the favour disposition during his stay in Hamburg (2001–2002). C.Froggatt and H.B.Nielsen would like to thank ITEP for inviting them to the XXXI-st ITEP Winter School where this paper was completed and L.Bergamin for interesting discussions.

This work was supported by the Russian Foundation for Basic Research (RFBR), projects 00-15-96786, 00-15-96562 and 02-02-17379.

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Figure captions

Fig. 1. (a) the scalar potential and (b) the superpotential for the simplest SUGRA model where the MPP conditions are satisfied. The standard supergravity mass units are used.

Fig. 2. The value of $\log [\Lambda_{SQCD}/M_{Pl}]$ versus $\log M_S$. The thin and thick solid lines correspond to the pure MSSM and the MSSM with an additional pair of $5 + \bar{5}$ multiplets respectively. The dashed and dash-dotted lines represent the uncertainty in $\alpha_3(M_Z)$. The upper dashed and dash-dotted lines are obtained for $\alpha_3(M_Z) = 0.124$, while the lower ones correspond to $\alpha_3(M_Z) = 0.112$. The horizontal line represents the measured value of $\Lambda^{1/4}$. The SUSY breaking scale is given in GeV.

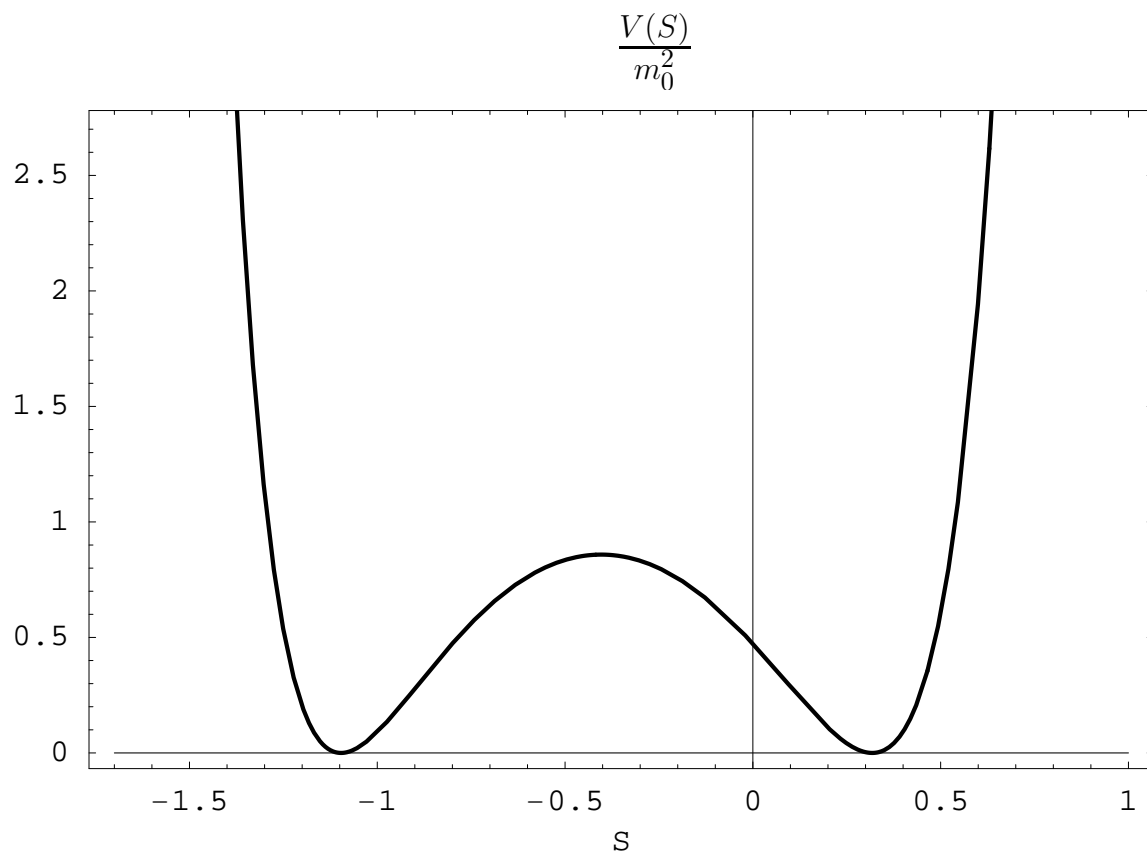


Fig.1a

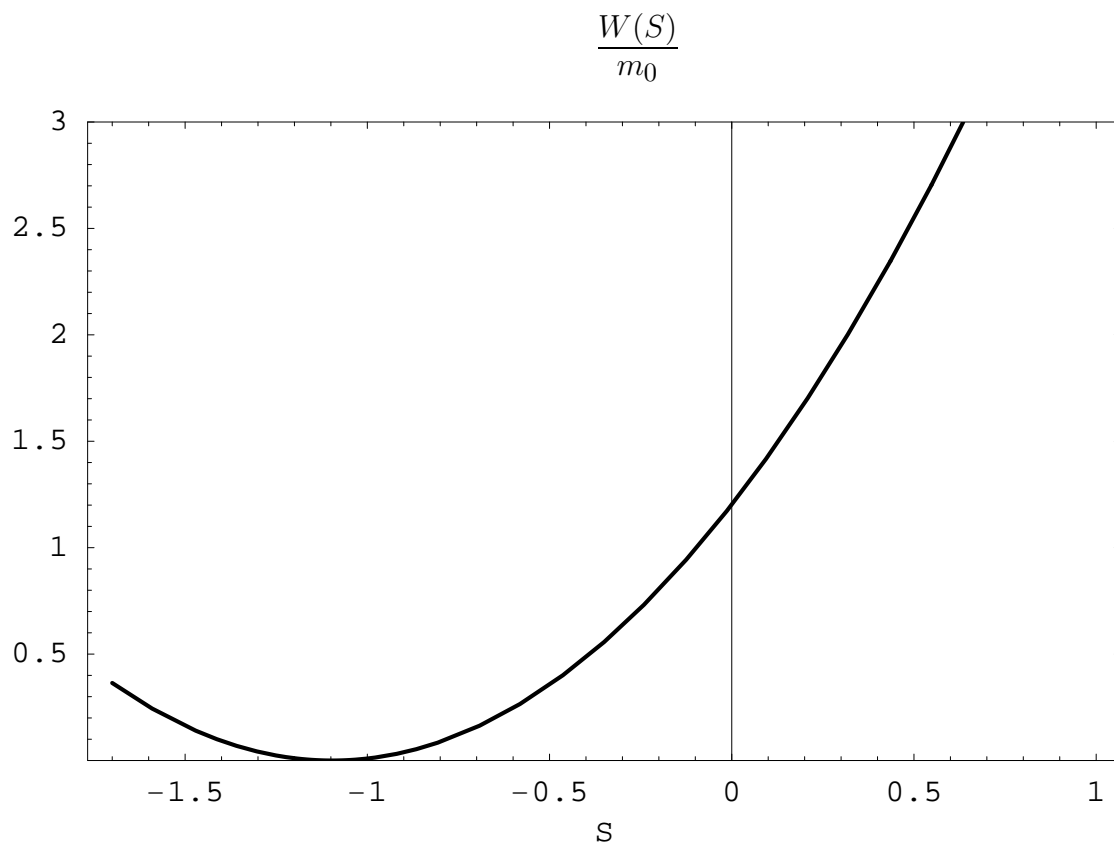


Fig.1b

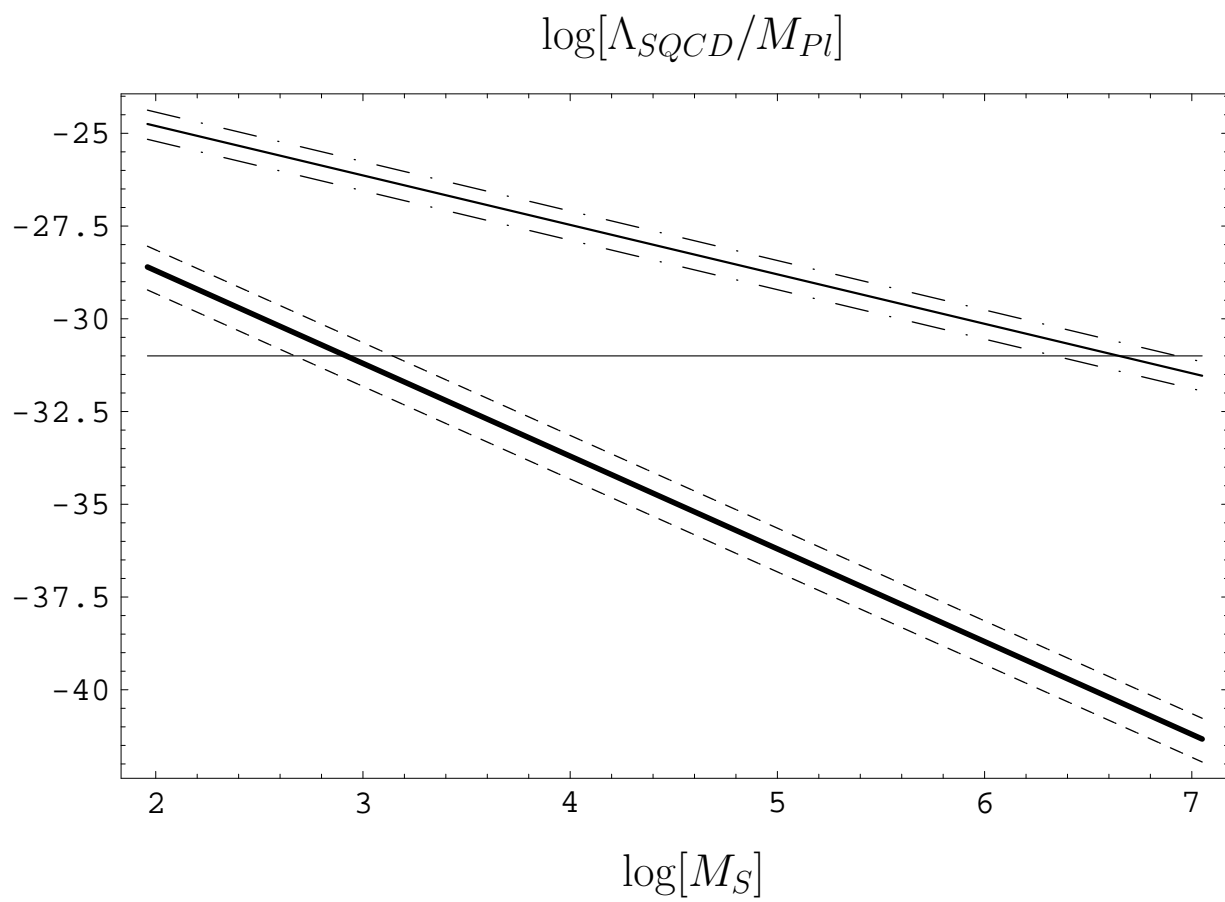


Fig.2